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PROPERTIES PRESERVATION OF EXPANSION OF MODELS OF NIP
THEORIES

ABSTRACT

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Actuality of the research theme.

At the present time one of the main directions of investigation in model theory, one of the branches of mathematical logic, is expansion of models by new relations.

The underlying theme in model theory is to classify first order theories. The first approximation in classifying was Shelah's notion of *stable theory*. Which recently was has broadened and now includes NIP theories.

Theory T is called to have the independence property (IP), if for some formula $\varphi(\bar{x}, \bar{y})$ in every model \mathfrak{M} of T for each $n < \omega$, there is a family of tuples $\bar{b}_1, \dots, \bar{b}_n$, such that for every subset I of n there exists a tuple $\bar{a}_I \in \mathfrak{M}$ such that $\mathfrak{M} \models \varphi(\bar{a}_I, \bar{b}_i) \Leftrightarrow i \in I$. If there is no such formula, then T is said to have non-independence property (NIP).

Important part of investigating complete theories is to study conditions on new relations necessary and/or sufficient to change class of model of complete theory in new signature or preserve it. One of the most significant classes of complete theories in NIP theories along with stable theories are o-minimal theories and a wider class including o-minimal theories - weakly o-minimal theories. Both of them are classes of totally ordered theories, such that for every model of these theories every definable subset is a finite union of either intervals for o-minimal theory or convex sets for weakly o-minimal theory. This classes of theories are in the main scope of exploration of this work.

Leading specialists in model theory, such as B.I. Zilber, E. Hrushovski, A. Neshin, B. Poizat, G. Cherlin, J. Baldwin, E. Bouscaren, A. Wilkie, Ch. Steinhorn, D. Macpherson, D. Marker, B. Baizhanov, S. Shelah, M. Benedikt, A. Pillay, have received profound results in different problems of expansions.

J.T. Baldwin and K. Holland found sufficient conditions for any ω_1 -categorical expansion of strongly minimal model by unary predicates to have a model complete theory. The field of real numbers, whose elementary theory admits quantifier elimination, is decidable and o-minimal and the expansion by a unary exponential function has a model complete and o-minimal theory by Alex Wilkie. B.S. Baizhanov proved that in some elementary extension the class of unary partial functions which are definable with parameters does not coincide with the class of unary partial

functions, definable with parameters in the initial language in this extension, if and only if an o-minimal expansion of a model of theory T which is dense o-minimal and admits quantifier elimination will be essential. Macpherson-Marker-Steinhorn proved that an expansion of weakly o-minimal structure by special kind of convex unary predicate preserves weak o-minimality. B.S. Baizhanov solved the problem of expansion of weakly o-minimal theories by unary convex predicate. B. Sh. Kulpeshov obtained a characterization of weakly o-minimal theories in terms of convexity of definable sets of one-types and introduced the notion of convexity rank. Thesis concerns the classification of expansion of weakly o-minimal theories of finite convexity rank which is quite new class of complete NIP theories.

Other direction of study is externally definable sets and expansions, which was introduced first by D. Macpherson, D. Marker and Ch. Steinhorn. Let $M = \langle M, \Sigma \rangle$, $N = \langle N, \Sigma \rangle$ be two structures of the signature Σ , such that $M < N$. We say that a set $A \subset M$ is externally definable, if $A = \varphi(N, \alpha) \cap M$ for some N -formula, $\alpha \in N \setminus M$. Macpherson-Marker-Steinhorn proved that any expansion of weakly o-minimal theory, such that all types are solitary, by unary convex predicate is externally definable weakly o-minimal

The aims and objectives of the study.

The work is devoted to study the questions of preservation of certain properties (weak o-minimality, quite o-minimality, countable categoricity, model completeness, convexity rank and others) under the expansion of models. The aims of the work are the following:

1. To investigate questions of preservation of certain properties of models by expanding unary predicates.
2. To investigate questions of preservation of certain properties of models by expanding equivalence relations.
3. To investigate questions of preservation of certain properties of models by expanding arbitrary binary predicates.

The main provisions for the defence of the dissertation:

1. Preserving both the countable categoricity and convexity rank when expanding a model of a countably categorical weakly o-minimal theory of finite convexity rank by a finite family of convex unary predicates.
2. Preserving both countable categoricity, convexity rank and quite o-minimality when expanding a model of a countably categorical quite o-minimal theory by a finite family of convex unary predicates.
3. Criterion for preserving both countable categoricity and weak o-minimality (and additionally the 1-indiscernibility) when expanding a model of a 1-indiscernible countably categorical weakly o-minimal theory of finite convexity rank by an equivalence relation partitioning the universe into infinitely many infinite convex classes.

4. Criterion for preserving countable categoricity for a weakly o-minimal expansion of a 1-indiscernible countably categorical weakly o-minimal theory of convexity rank 1 by an arbitrary binary predicate.

5. Criterion for preserving countable categoricity for a weakly o-minimal expansion of a non-1-indiscernible countably categorical weakly o-minimal theory of convexity rank 1 by an arbitrary binary predicate.

6. Preserving weak o-minimality when expanding a weakly o-minimal ordered group by an externally definable binary predicate.

The objects of research are complete NIP theories (theories without the independence property) and their classes of models. In particular, NIP theories include stable theories and weakly o-minimal theories.

The research subjects models of NIP theories, their properties and properties of these models under expansion by unary or binary predicates or equivalence relations.

Research methods Includes methods of Classical Model Theory (for example, method of quantifier elimination), including ones which have been developed in model theory in the 1980's and later. Among them we can note the methodology of studying ordered structures, based on such notions as o-minimality and variants of o-minimality. In such cases it is typical to apply strict restrictions on sets definable by a formula with one free variable. Thus, an o-minimal structure M can be considered as an L -structure, where $L \supset L_0 = \{<\}$, $<$ is a linear order on M , and every definable subset of the structure M is quantifier-free L_0 -definable. It gives a template for other definitions: replace L_0 by some other unknown language, consider L -structures that the L_0 -reduct is of stipulated type (e.g. a linear order), and we demand that every definable subset of the structure M is (quantifier-free) L_0 -definable (we can ask this for all models of this theory). Aside from that, we can note the methods of studying ordered structures developed in the last 20 years, such as describing models through analysis of behaviour of definable unary functions, the study of models via classification by convexity rank and others.

Scientific novelty of the dissertation research. Properties preservation of expansion of models of complete theories such as NIP theory problem is open at the present time. Classes of theories under the study have not been investigated on a considered expansions.

Theoretical and practical significance of the research. Researches in this area constitute steps in classification of complete NIP theories. Expected results on the nature of expansions can be applied to group, ring and field theory

Connection of the dissertation thesis with the other scientific research works. The dissertation thesis was implemented within the scientific projects of the program of grant financing of fundamental researches in the area of natural sciences of the Ministry of education and science of the Republic of Kazakhstan "Properties of types in dependent theories" (2015-2017 years, 5125/GF4), "Basic and derived objects for ordered and generating structural objects as well as elementary theories" (2018-2020 years, AP05132546) and "Conservative extensions, countable ordered models and closure operators" (2018-2020 years, AP05134992).

The work approbation. Results of the work were presented and discussed at many foreign and domestic international scientific conferences and seminars:

The Sixth Congress of the Turkic World Mathematical Society (Astana, 2017);

The Twelfth International Summer School-Conference "Problems Allied to Universal Algebra and Model Theory" (Erlagol, Russia, 2017);

International Conference "Mal'tsev Meeting" (Novosibirsk, Russia, Institute of Mathematics, 2017, 2018);

ASL European Summer Meeting "Logic Colloquium" (Stockholm, Sweden, 2017; Udine, Italy, 2018);

ASL North American Annual Meeting (Macomb, USA, Western Illinois University, 2018);

The Sixth World Congress and School on Universal Logic (Vichy, France, 2018);

The Sixteenth Asian Logic Conference (Nur-Sultan, 2019);

Annual International April Mathematical Conference (Almaty, Institute of Mathematics and Mathematical Modeling, 2017, 2018, 2019, 2020).;

Scientific seminars of the department of algebra and mathematical logic of the Institute of Mathematics and mathematical modeling;

Results of the dissertation work were discussed with prominent model theory specialists during the scientific training at the University of Leeds at Leeds, UK.

Assessment of the completeness of the aims of the work. All the results are new and are based on our own methods and tools. Conditions of preservation of either weak o-minimality, countable categoricity under expansion by unary or binary predicates were found. Therefore, the work's objectives were fully completed.

Suggestions on applications of the obtained results. The results obtained in this area of model theory can be used during the study of models of NIP theories, particularly expansions of weakly o-minimal theories. Results on the expansions by externally definable sets can be applied to theories of algebraic structures.

Assessment of scientific level of the work in comparison with the achievements in the scientific direction. The results obtained in comparison with the best achievements of foreign colleagues do not lose and contribute to the study of expansion of models of complete theories.

Publications. The main results of the dissertation were published in 20 works, including 3 articles published in journals having a non-zero impact factor according to international databases Web of Science and (or) Scopus; 5 papers published in domestic journals recommended by Committee for Control in the Field of Education and Science of Ministry of Education and Science of the Republic of Kazakhstan, and 12 abstracts in materials of international scientific conferences.

Volume and structure of the dissertation. The work includes the title page, contents, normative references, definitions, notations and abbreviations, introduction, 5 sections, conclusion and references. Total volume of the dissertation is 76 pages, the work contains 4 illustrations and 101 literature references.

Main content of the dissertation work. The introduction of the thesis includes actuality of the research theme, the aim of the work, the objectives of the work, the

main provisions for the defence of the dissertation, the object of research and research subject, methods of research, scientific novelty and theoretical and practical significance of the dissertation thesis, connection of the dissertation thesis with the other scientific research works, approbation of obtained results, author's publications, as well as volume, structure and content of the dissertation thesis.

The first section explains historical background and the current state of the investigated area of model theory.

The second section of the dissertation gives preliminary information and considers expansions of models by unary predicates.

The third section is devoted to expansions by equivalence relations of countably categorical, weakly o-minimal theories. Found a criterion for preserving countable categoricity and weak o-minimality.

In the fourth section of the dissertation considers arbitrary binary expansions of models of 1-indiscernible and non-1-indiscernible countably categorical, weakly o-minimal theories of convexity rank 1.

The fifth section is focused on the class of externally definable expansions in the scope of preserving model completeness.

To clarify the main result of the section we also show different examples of expansions which does not preserve certain properties.

The conclusion lists and generalizes the main results obtained during the implementation of the dissertation thesis.